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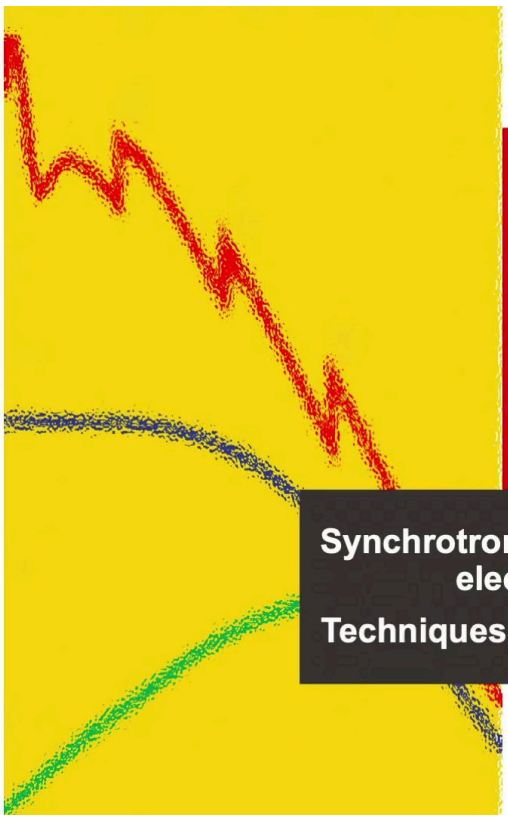


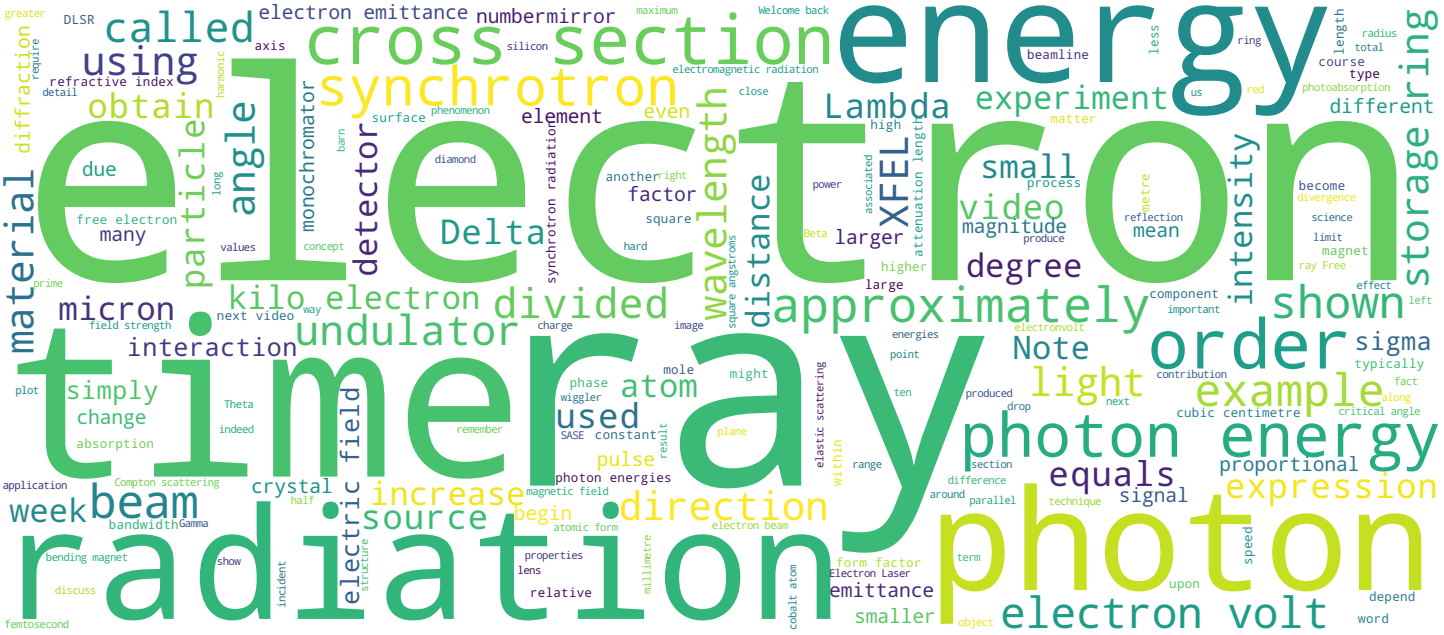
Interactions of x-rays with matter – introductory comments and insights (Part 1)

Synchrotrons and x-ray free-electron lasers

Techniques and applications

École polytechnique fédérale de Lausanne

Prof. Philip Willmott



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Video

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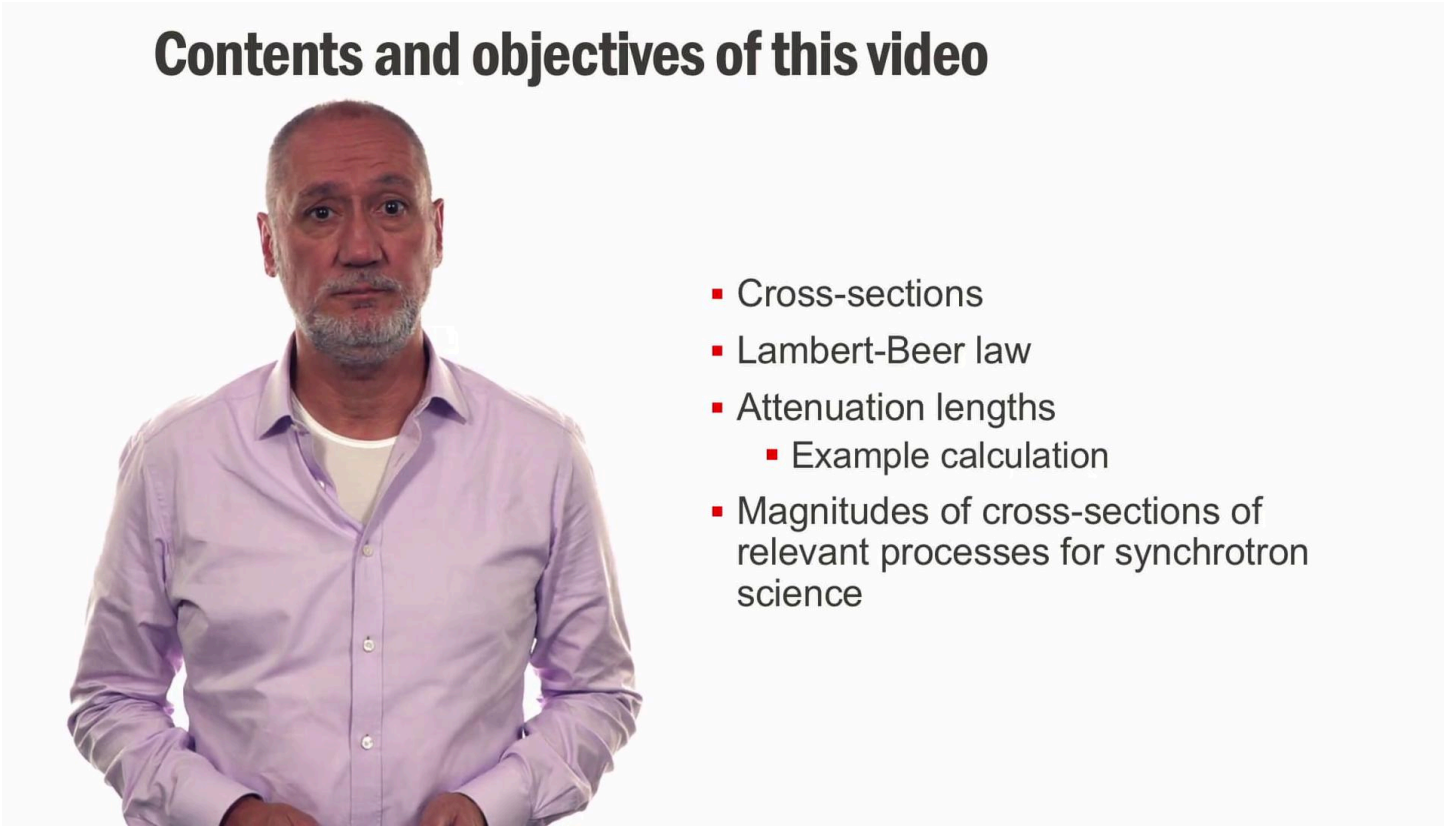


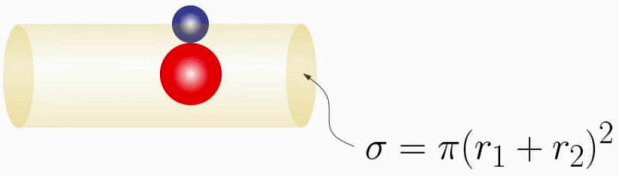
Hello, and welcome back to week two of this course, Synchrotrons on X-ray Free Electron Lasers, Techniques, and Applications. This week, we will consider the interaction of X-rays with matter, beginning with the definition of cross-sections and the Beer-Lambert law, following with the concept of dipole radiation and continuing with a detailed explanation of the so-called atomic form factor. This leads us to the phenomena of elastic scattering, refraction, reflection, and absorption. For the sake of completeness, we will also briefly consider Compton scattering important for experiments using very high photon energies. In this first video, we look at cross-sections as a measure for the strength of interaction of X-rays with matter via different processes.

Notes

Summary

0m 05s

Cross-sections



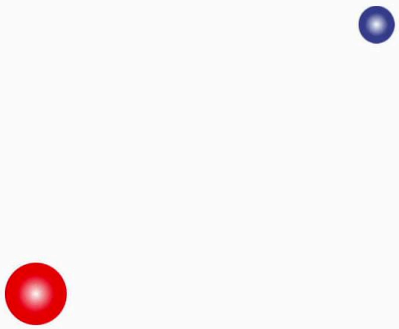
The simplest description of cross-sections is to consider the interacting particles to be hard spheres, much like billiard balls. If the closest distance between two particles is greater than the sum of their radii, no interaction occurs. If the distance is smaller than this limit, however, they will interact. It's thus apparent that the cross-section for interaction is sigma is equal to Pi times the sum of the two radii, r1 plus r2 squared.

Notes

Summary

1m 01s

Cross-sections



(2) Action at a distance (e.g. electromagnetic radiation interacting with atoms/matter)

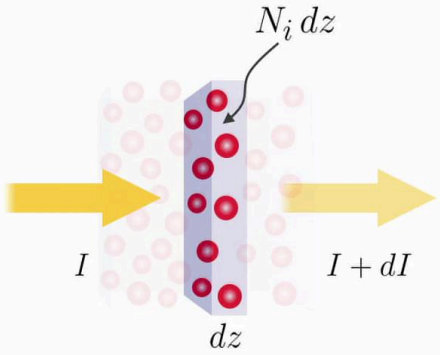
More commonly, and certainly in the case of interaction of electromagnetic radiation with matter, action can happen at a distance. The effective cross-section can be calculated by integrating across all possible separations.

Notes

Summary

1m 39s

Beer-Lambert law



$$dI = -N_i \sigma I dz$$

$$\Rightarrow \frac{dI}{I} = -N_i \sigma\, dz$$

$$I = I_0\, e^{-N_i \sigma z}$$

Now, let's consider a cubic collection of interacting particles with the density of Ni per unit volume. We concentrate on a thin slice of this of thickness dz, where we'll have made dz sufficiently small that no particle will shadow another in the direction of dz. The aerial density of particles in this slice is simply Ni times dz per unit area. A beam of photons instant on this slice with an intensity I will be reduced by an amount dI after passing through the thin slice. Clearly, dI needs to be a negative term in order for the intensity to drop. Each particle has a cross-section sigma, hence the loss in intensity is DI is equal to minus Ni dz times sigma times the incident intensity I. Rearranging this, we get DI over I is equal to minus Ni sigma dz. We can integrate this expression to obtain the Lambert-Beer equation for exponential decay, I is equal to I0 multiplied by e to the minus Ni sigma z, where I0 is the initial intensity of the beam.

Notes

Summary

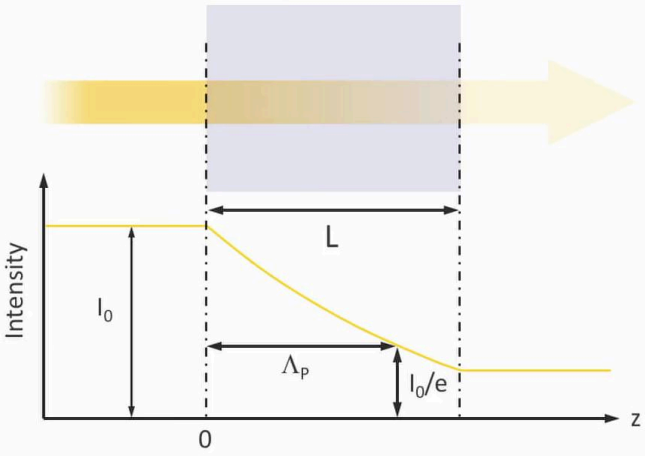
1m 58s

Cross-sections and attenuation lengths

- Process P
- Cross-section σ_P
  - Units: area/atom for process P
- Euler's number $e^1 = 2.71828\ldots$
  $e^{-1} = 0.3679\ldots$
- $I = I_0 \exp(-N_i\sigma_P z) = I_0 \exp(-z/\Lambda_P)$
- Λ_P = attenuation length for process P

$$\sigma_P = (\Lambda_P N_i)^{-1}$$

N_i = number density of atoms



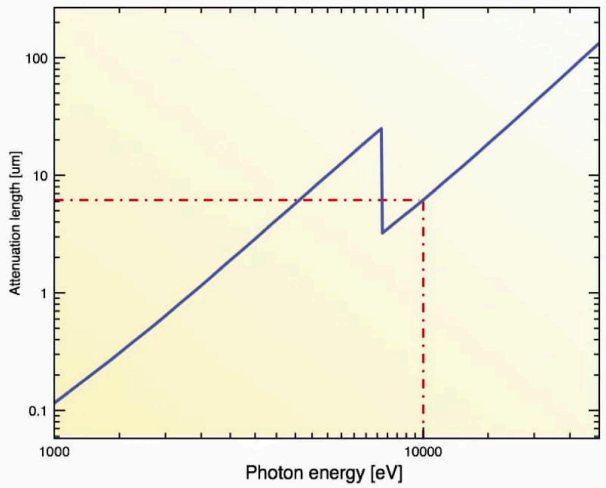
For a given process P, there will be a cross-section sigma P. The irrational number e named Euler's number is approximately equal to 2.718. Its inverse, 1 upon e is approximately 0.3679. From our expression for the Beer-Lambert law, we immediately see that if Ni sigma Pz equals 1, the initial beam of radiation will have been attenuated to I0 upon e or 0.3679 of I0. Now, Ni and sigma P depend only on the material, so it is the value of z, which determines when the beam has been attenuated to 1 upon e of its initial value. This value of z we call the attenuation length which for process P, we label as lambda P. Hence, Ni sigma P lambda P equals 1,. and so sigma P is equal to 1 upon lambda P and I. We show this graphically here.

Notes

Summary

3m 34s

Example: photoabsorption of cobalt @ 10 keV



Data from CXRO http://henke.lbl.gov/optical_constants/

- Cobalt metal
  - Density $\rho = 8.90$ g cm^{-3}
  - Molar mass = $M_W = 58.933$ g mol^{-1}
  - $\Rightarrow N_i = N_A \rho / M_W = 9.0943 \times 10^{10}/\mu m^3$
- Photoabsorption attenuation length @ 10 keV = $\Lambda_a = 6.2$ μm
- Cross-section = $1/(N_i \Lambda_a)$

  $\sigma_a = 1.774 \times 10^{-12}$ μm^2
  $= 1.774 \times 10^{-4}$ Å^2
  $= 1.774 \times 10^{4}$ barn (1 barn = 10^{-28} m^2)

Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

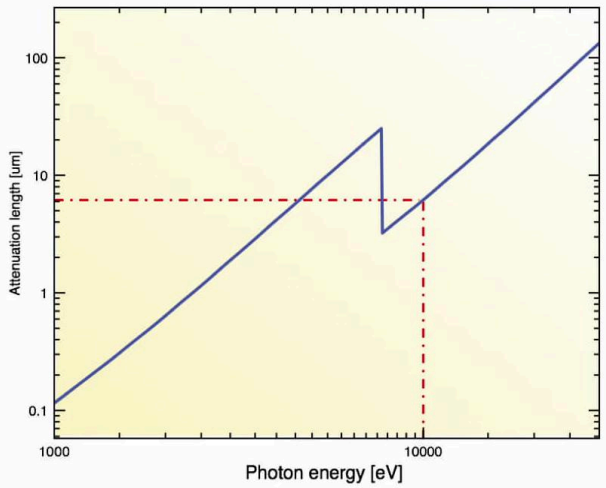
Let us consider a practical example of the case of the photoabsorption of cobalt metal at 10 kilo electron volts. We begin by calculating Ni, the number density of cobalt atoms. The mass density of cobalt is 8.9 grams per cubic centimetre, and its molar mass is 58.933 grams per mole. There are therefore 8.9 divided by 58.933 moles per cubic centimetre, which is equal to 0.151 moles per cubic centimetre. Therefore, the number density is this times Avogadro's number 6.022 times 10 to the 23 particles per mole, which is equal to 9.09 times 10 to the 22 cobalt atoms per cubic centimetre, or 9.09 times 10 to the 10 cobalt atoms per cubic micron. I converted from cubic centimetres to cubic microns because in the next step, the attenuation length is most commonly given in microns. We obtained the attenuation length from the CXRO website, the link to which is given here. At 10 kilo electron volts, the attenuation length is 6.2 microns. From this, we obtain a cross-section of 1.774 times 10 to the minus 12 square microns per atom. This is a pretty tiny number. Converting to square angstroms, we obtain a cross-section of 1.774 times 10 to the minus 4 square angstroms, still rather puny.

Notes

Summary

4m 54s

Example: photoabsorption of cobalt @ 10 keV



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  σ_a = 1.774 x 10^{-12} μm^2
  = 1.774 x 10^{-4} Å^2
  = 1.774 x 10^4 barn (1 barn = 10^{-28} m^2)

Avogadro's number N_A = 6.022 x 10^{23} mol^{-1}

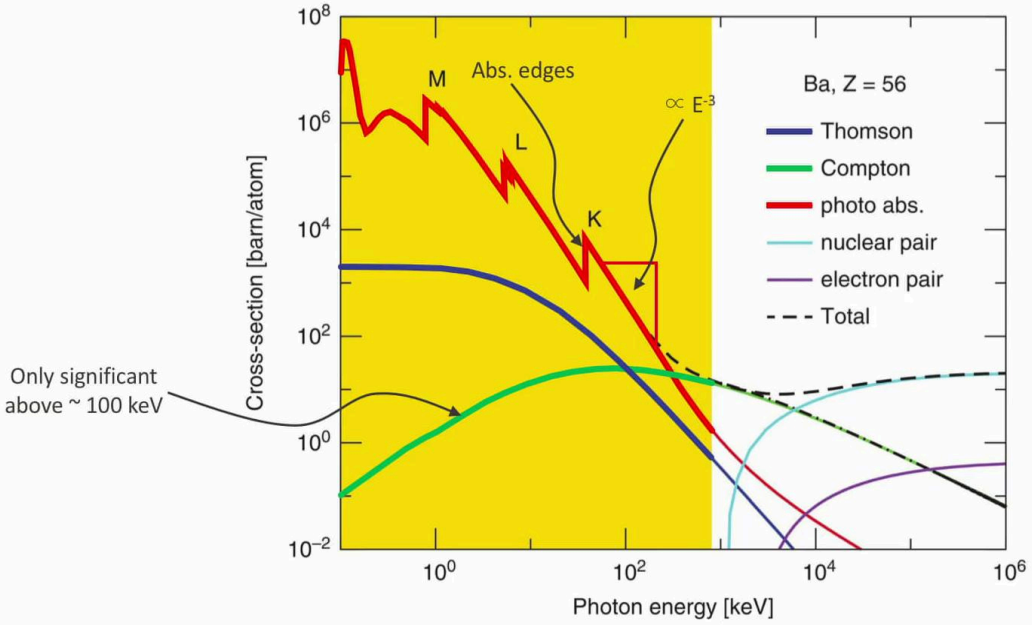
A commonly used unit of cross-sectional area is the so-called barn, which is 10 to the minus 28 square metres or 10 to the minus 8 square angstroms. Hence expressed in barns, the cross-section for photoabsorption at 10 kilo electron volts for cobalt is 17,740 barns per cobalt atom.

Notes

Summary

6m 37s

Interaction strengths of UV and x-rays with matter



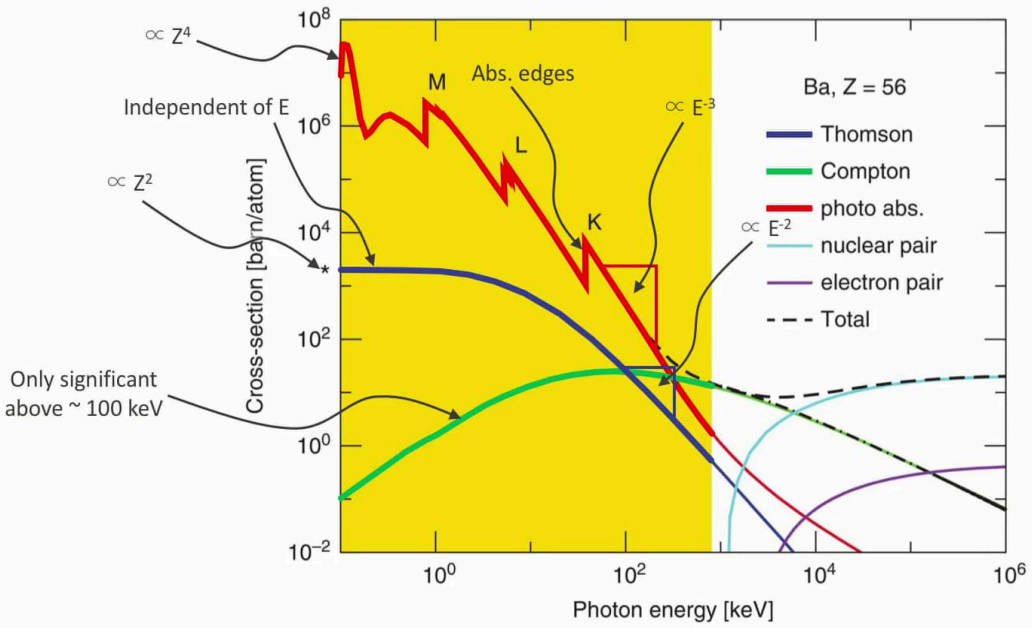
We finish this introduction to cross-sections by revisiting our plot of the relevant processes regarding the interaction of synchrotron radiation with matter up to approximately 1 mega electron volt, the yellow shaded part of the graph for the case of barn. We begin by getting Compton scattering out of the way. This describes inelastic scattering of light by free charged particles and requires that the light be described as a particle or photon. It's only significant above approximately 100 kilo electron volts, but must be considered for certain high energy X-ray experiments. Other than a brief description of the physics behind Compton scattering later in this week's videos, we shall not return to this phenomenon, so won't dwell on it further here. In general, photoabsorbtion has the largest interaction strength. It is characterised by abrupt increases at photon energy's equal to the binding energy of electrons in the atom called absorption edges, which are labelled K, L, M, et cetera, starting from the highest energy edge and proceeding down in photon energy. Other than this, the cross-section drops off as the inverse cube of the photon energy, as is evident here by the gradient of minus 3 in this double logarithmic plot.

Notes

Summary

7m 02s

Interaction strengths of UV and x-rays with matter



The absolute value of the photoabsorbtion cross-section is strongly dependent on the atomic number being proportional to the fourth power of z. The elastic scattering or Thomson cross-section is in general significantly weaker than that for photoabsorption. Above approximately 2 or 3 kilo electron volts, it drops off as the inverse square of the photon energy. Below this limit, it gradually flattens off to a constant value with respect to the photon energy, but one that is proportional to the square of the atomic number z. We look at these dependencies and the physical reasons behind them in the next videos. First, however, we need to examine the response of a free electron to electric fields and, in particular, electromagnetic radiation, which we now do in the next video.

Notes

Summary

8m 42s